Let $2x=x'$, then

$$\int_0^{\frac{1}{2}\pi}\log\sin 2x\,dx=\tfrac{1}{2}\int_0^{\pi}\log\sin x'\,dx'=\int_0^{\frac{1}{2}\pi}\log\sin x\,dx.$$

$$\therefore 2u=u-\frac{\pi}{2}\log 2,\quad u=-\frac{\pi}{2}\log 2=\frac{\pi}{2}\log\tfrac{1}{2}.$$

Also solved by M. E. Graber, G. W. Greenwood, L. E. Newcomb, and G. B. M. Zerr.

206. Proposed by DR. O. E. GLENN, Drury College.

Evaluate $\int_0^1(1-z^n)^m\frac{\partial}{\partial z}\log(1-z^nx^n)dz$, assuming $-1<x^n<+1$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$u=-\int_0^1\frac{nx^nz^{n-1}(1-z^n)^m}{1-x^nz^n}dz$. Let $1-z^n=y$, then we have

$$u=-\int_0^1\frac{x^ny^m}{1-x^n+x^ny}dy=-\int_0^1\frac{y^m}{y+a}dy,\text{ where }\frac{1-x^n}{x^n}=a.$$

$$\therefore u=-\int_0^1\left(y^{m-1}-ay^{m-2}+a^2y^{m-3}.....(-1)^{m-1}a^{m-1}+\frac{(-1)^ma^m}{y+a}\right)dy$$

$$=-\left.\frac{y^m}{m}-\frac{ay^{m-1}}{m-1}+.....+(-1)^{m-1}a^{m-1}y+(-1)^ma^m\log(y+a)\right]_0^1$$

$$=-\left[\frac{1}{m}-\frac{a}{m-1}+.....+(-1)^{m-1}a^{m-1}+(-1)^ma^m\log\left(\frac{1+a}{a}\right)\right]$$

$$=-\left[\frac{1}{m}-\frac{(1-x^n)}{x^n(m-1)}+\frac{(1-x^n)^2}{x^{2n}(m-2)}+.....\frac{(-1)^{m-1}(1-x^n)^{m-1}}{x^{(m-1)n}}\right.$$

$$\left.+(-1)^{m+1}\frac{(1-x^n)^m}{x^{mn}}\log(1-x^n)\right].$$

DIOPHANTINE ANALYSIS.

128. Proposed by F. P. MATZ, Ph. D., Sc. D., Reading, Pa.

Required the highest powers of 2, 3, 5, 7, contained in (1000)!

I. Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

$$(1000)!=2^{500}(500)!(1.3.5........999)$$
$$(500)!=2^{250}(250)!(1.3.5.........499)$$

....

Proceeding thus we find the powers required are

$$2^{994},\quad 3^{498},\quad 5^{249},\quad 7^{164}.$$